

Conceptual Ternary Diagrams for Shape Perception: A Preliminary Step

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Abstract

This work-in-progress provides a preliminary cognitive investigation of how the external visualization of the ‘Ternary diagram’ (TD) might be used as an underlying model for exploring the representation of simple 3D cuboids according to the theory of Conceptual Spaces. Gärdenfors introduced geometrical entities, known as conceptual spaces, for modeling concepts. He considered multidimensional spaces equipped with a range of similarity measures (such as metrics) and guided by criteria and mechanisms as a geometrical model for concept formation and management. Our work is inspired by the conceptual spaces approach and takes ternary diagrams as its underlying conceptual model. The main motivation for our work is twofold. First, Ternary Diagrams are powerful conceptual representations that have a solid historical and mathematical foundation. Second, the notion of overlaying an ‘Information-Entropy’ function on a ternary diagram can lead to new insights into applications of reasoning about shape and other cognitive processes.

Introduction

People effortlessly represent and reason about shapes. They are able to use previous experiences of objects to make effective judgments about the properties of new shape related concepts and shape similarities. To model the formation and behaviour of concepts Gärdenfors introduced conceptual spaces (2000). Conceptual spaces are essentially multi-dimensional (vector) spaces equipped with similarity measures. Conceptual spaces (CS) are useful in modeling concepts and knowledge because they are richer representations than predicate calculus; conceptual spaces are not based on arbitrary strings but judgments of similarity and categorization in a geometrical space.

Conceptual spaces have been useful in explaining conceptual phenomena and in constructing concepts for

intelligent agents. Determining the dimensions and nature of the similarity measure can be challenging. According to Gärdenfors, a fundamental methodological question that one needs to ask in the construction of a conceptual space is:

Which are the best dimensions, the best topology [and] the best metric to be used for representing a particular cognitive process? (Gärdenfors 2000, p. 260)

To explore such broad questions, we consider the topology of the 2-simplex (or equilateral triangle) commonly known as the Ternary Diagram (TD). TDs have been used in scientific applications for more than 200 years. Here we give an initial cognitive interpretation to TD by borrowing from the geological discipline of grain shape analysis. Our Conceptual Ternary Diagram (CTD) possess some interesting mathematical properties which might eventually be used in the cognitive process of similarity judgments.

Whilst we acknowledge that our preliminary work is limited to representing 3D cuboids and thus we neglect many experimentally investigated aspects of human shape perception and recognition, such as surface contours, texture, etc (Todd 2004) we believe the advantages of considering such an approach as an underlying model for a conceptual space reside in the visual-spatial nature of the TD as an external visualization. (Hegarty 2004)

First, we provide an overview of Gärdenfors’ conceptual spaces approach. Second, we introduce the TD as a means to describe combinations. Third, we show how a TD can be construed to be a CS to produce a CTD. Fourth, we briefly discuss the potential of using entropy metric within a CTD. Lastly, we will present a number of useful lines of future enquiry.

Conceptual Spaces

According to Gärdenfors (1997), there exists a *conceptual* level of representing human thought which complimentary bridges the gap between the dominant connectionist and symbolic levels of representing an agent's cognition.

In the theoretical framework of *Conceptual Spaces*, he proposes that such a representation level can be modelled according to principles which are of geometric nature (Gärdenfors 2000). Broadly, the theory involves describing how *objects* are represented in some representational space.

The first step in the construction of a conceptual space is selecting suitable *quality dimensions*. Examples of quality dimensions include temperature, weight, or the three spatial dimensions of height, width and depth. A general assumption is that such quality dimensions possess a geometric structure. If these quality dimensions are deemed to be psychologically integral (Garner 1974) then the geometric structures can form a *domain*.

Objects are *points* in the domain and concepts are *regions*. The specific *properties* of an *object* can be shown to depend on the dimensions of the subspace of a *region*. For example, a red car will lie within the red region of the car colour subspace. A key notion in the conceptual framework is convexity. Convex regions in a conceptual space correspond to natural kinds. Natural kinds are important classes because they tend to be robust to certain types of reasoning such as generalization (Rosch 1975).

Together with several mathematical/geometric criteria, a concept is represented by considering the relationships between several domains, their associated regions and quality dimensions. An example of how one might possibly represent the concept of "apple" using discrete labels is given in the table below.

| Concept | Domain | Region | Quality Dimension |
|---------|-----------|---------------|-------------------|
| 'Apple' | Color | Red-Green | Wavelength |
| | Shape | Roundish | Shape metric |
| | Texture | Smooth | Various |
| | Taste | Sweet/Sour | |
| | Fruit | Seed Type etc | |
| | Nutrition | Fiber etc. | |

Table 1: Concept of 'Apple', Adapted from (Table 4.1 Gärdenfors 2000, p. 103)

Considering the work in showing the algorithmic feasibility of such a CS approach in reasoning about general categories (Gärdenfors & Williams 2001) and the overlapping geometric-shape theme, little attention has been paid to the CS approach of modeling shape concepts. One notable exception is the work of Chella and colleagues, whom applied CS to the field of high level computer vision (Chella, Frixione & Gaglio 1997). Their

work involves describing shape objects ('knoxels') according to the quality dimensions specified by constructive solid geometry and formalized by the mathematics of super quadratics (Chella, Frixione & Gaglio 2001). In arguing that their approach compares favorably to related computer vision frameworks, they acknowledge that the '[assumption of] the existence of a 3D internal representation of concepts' is subject to widespread ongoing debate. They also suggest that problems applying the theory of conceptual spaces to their given field of vision lie in choosing the appropriate 1) quality dimensions 2) similarity measures.

To help relax the '3D representation assumption' and attend to such dimension and metric selection problems of the constructing a CS, we turn to the TD as an external visualization.

Ternary Diagrams as an External Visualization

TDs provide a concise description of combinations. They can be considered as external visualizations which might be seen to augment internal cognition (Hegarty 2004).

The visual and geometrical *form* of a standard TD is that of an equilateral triangle, or mathematically known as a low dimensional member of the larger simplex family (Weisstein 2009).

At the vertices of the triangle it is common practice to provide *labels* that identify the components being studied. Furthermore, appropriate *scales* are provided along the sides of the triangle. Finally, to assist with plotting, it is normal for a *grid* scale to be overlaid. An example of a standard Ternary Diagram is shown below in Figure 1.

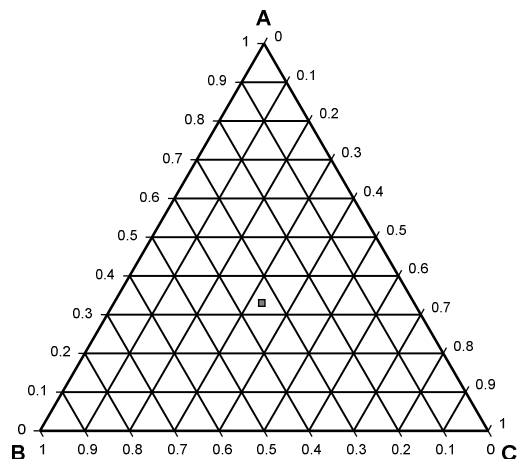


Figure 1: The standard visual form of a Ternary Diagram. Showing a,b,c = 0.3, 0.3, 0.3 point (Prosim 2004)

In a careful review of the historic uses of TDs, it becomes apparent that the natural strength of such a technique is in its ability to support people as they visually represent and

reason about compositions and mixtures. Two prominent examples are Young/Maxwell’s theory of colour mixing, and Gibb’s formulation of liquid phase laws (Howarth 1996).

An early formalism of this reasoning was given by the Italian statistician Bruno De Finetti, in his studies of the composition of genotypes (Rossi 2000). In following De Finetti’s formalism for our own preliminary work, a TD is suited to studying systems of three components (A,B,C) provided that the proportions of such components (a,b,c) are positive, linearly dependant and sum to a nonzero constant, K, that is,

$$a + b + c = K$$

The constant K, is usually normalized to 1, or 100%. A more rigorous statistical treatment, which deals with zero components and different types of dependencies is provided by Aitchinson (1982).

An important area where these underlying assumptions are met is the geological discipline of sedimentology. In this field TDs are used to describe the composition of grain shapes in a given soil sample. They do so by treating the shape as a 3-component system long, intermediate and short axes (Sneed & Folk 1958). In such an analysis, the De Finetti ‘sum to constant’ requirement is followed such that the long, intermediate and short axial lengths to add to a *Unitary* whole – a single pebble.

$$L + I + S = 100\%$$

However, due to the physical nature of the system, the axial lengths have the further constraints placed on them of ‘Relative’ and ‘Perpendicular’ lengths.

- Relative Axial Lengths $L \geq I \geq S$
- Perpendicular $L \perp I \perp S$

The constraints are used to assign a ‘shape metric’ that is subsequently used in computational models to predict the behavior of grains (for example, the location of a given type of precious metal sediment along a river bed.)

Whilst several shape metric descriptors exist in the grain shape literature, (for recent comprehensive reviews see (Blott & Pye 2008; Le Roux 2005)) The ‘Shape Entropy’ function (Hofmann 1994a) is of particular interest to CTDs, which will be covered in subsequent sections.

Ternary Diagram as a Conceptual Space

In encouraging the adoption of his CS framework, Gärdenfors offers the following:

There is, in general, no unique way of choosing a dimension to represent a particular quality but a wide array of possibilities (Gärdenfors 2000, p. 7).

A grain shape TD can be interpreted as a CS. When considered within the context of CS, one can treat ‘grains’ as simple cuboid shaped objects, which are described by the quality dimensions of long, intermediate and short ‘axial lengths’. A domain can then be constructed which due to the specified constraints, takes triangular geometric form. An example of such a CTD with quality dimensions, a domain of cuboid shapes, a region of concepts and points of objects, is presented in the figure below.

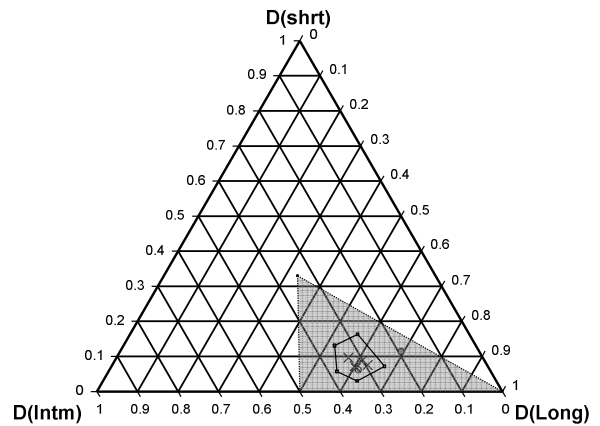


Figure 2: Our preliminary Conceptual Ternary Diagram (CTD)

A clearer idea of how shape objects would be represented in such a domain can be seen in Figure 3 below. Where a pictorial overlay has been provided by Hofmann in his study of grain shapes (1994b).

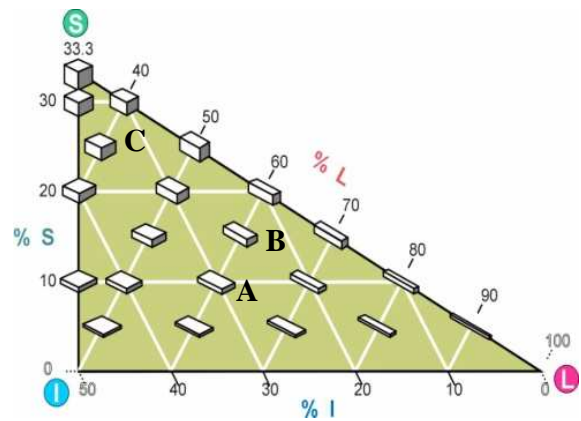


Figure 3: Domain of 3D regular polygons, ‘Box, Sheet and Stick’ Shapes Source: (Hofmann 1994b) reused with permission

It can be seen from a simple observation of the labelled boxes in Figure 3 above, that by presenting a pictorial overlay of shape objects, CTD has a similarity measure which is inherently built into the visual-spatial form of the diagram, e.g. a simple distance measure can be used to determine that object A is more similar to object B than to object C on the basis of proximity in the diagram– see Figure 3.

'Entropy' in a Conceptual Ternary Diagram

Hofmann (1994a) proposes that the axial conditions describing grain shape can be interpreted as the 'relative information entropy' function of Shannon. (Shannon 1949)

In an exploratory proposal, Hofmann draws upon earlier geological work (Pelto 1954) in proposing a 'relative axial uniformity' metric of the form:

$$H_{GShape} = -100 \cdot \frac{(p_{Long} \cdot \ln p_{Long}) + (p_{Intm} \cdot \ln p_{Intm}) + (p_{Short} \cdot \ln p_{Short})}{H_m}$$

Where p_{long} , p_{int} and p_{short} are the relative proportions of the axial lengths of grains, and H_m is the maximum relative entropy, $\ln 3$ (1.0986), for a three-axial system. The Relative Axial Uniformity ('shape entropy') is a maximum (1) when all three axes are equal and decreases in concentric intervals as can be seen as dark lines and the hypotenuse scale of Figure 4 below.

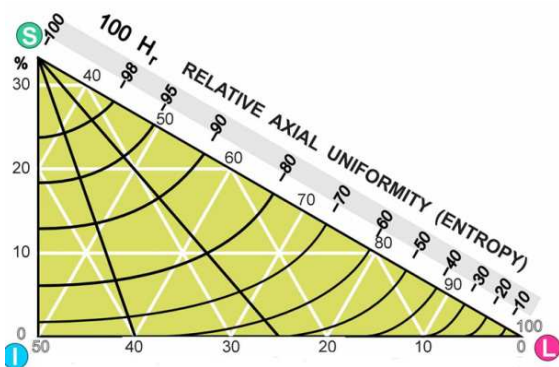


Figure 4: Relative Entropy in a CTD. Source: (Hofmann 1994b)

Such an entropy function is closely related to the well established and mathematical proven 'divergence' result of Kullback and Leibler (1951). We are exploring how entropy represented in this way can be used in the conceptual spaces setting. Given cognitive conceptual spaces have a strong self-organizational formation we expect to find evidence of entropy relationships at work in concept formation. The domain we will use for this work is shape because of the relatively transparent relationship between shape perception and concepts.

Discussion

Why consider the TD in the first place?

An obvious, but nonetheless important visual spatial property of CTDs is that they are able to present 3D dimensions using a 2D description whilst preserving the proximity relationships between objects.

To fully understand this and other visual spatial properties, one would need to undertake a comprehensive study of the cogno-psychological (Cheng, Lowe & Scaife 2001;

Cleveland & McGill 1985) and visual processing (Chabris & Kosslyn 2005) aspects. Whilst such a study would be beneficial, and to the best of the authors knowledge is lacking, it is beyond the scope of this preliminary work.

Limitations and Advantages

We have focused on simple shapes which can be described using three components. Aspects of complex shapes (curves, multiple components), optical structure (shading/luminance, surface texture/contours, motion, binocular disparity, and viewing angle) have been abstracted away. For a recent concise review of the state of the art in modeling shapes see Todd (2004).

Whether other quality dimensions can be identified which allow a broader application of such a TD representation to cover more complex shapes (i.e. 'cylinders') remains a question for further studies. An interesting starting point might be to explore whether the 'knoxel' approach of Chella's (2001) might be made to align with our conceptual ternary diagram representation.

On Entropy

Since entropy plays an important and meaningful role in all physical systems, it might prove interesting to further develop the mathematical definitions and cognitive interpretation of such an entropy metrics in CTDs. If seen to be mathematical sound and experimentally proven, the speculative question then arises – What is the role of entropy in concept formation or other cognitive processes?

Questions for future research

This early work in progress presents many opportunities for discussion, and raises the following research questions:

Are there merits in pursuing this transdisciplinary approach? – Can we apply other modeling techniques from the physical sciences to the cognitive sciences?

Is there evidence of previous use of TD-like representations in the Cognitive shape processing community?

How can we experimentally explore the CTD as an external visualization in modeling similarity? Can the similarity of box, sheet and sticks be successfully modelled according to the Kullback-Leibler divergence?

How can we overcome the initial limitations of such a visualization allowing for the representation of more complex shapes? For example, how might we represent the context and the salience of more complex shapes with respect to an agents' purpose?

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References

- Aitchison, J. 1982, 'The Statistical Analysis of Compositional Data', *Journal of the Royal Statistical Society. Series B (Methodological)*, vol. 44, no. 2, pp. 139-177.
- Blott, S.J. & Pye, K. 2008, 'Particle shape: a review and new methods of characterization and classification', *Sedimentology*, vol. 55, no. 1, pp. 31-63.
- Chabris, C.F. & Kosslyn, S.M. 2005, 'Representational correspondence as a basic principle of diagram design', in S.O. Tergan & T. Keller (eds), *Knowledge and Information Visualization: Searching for Synergies*, vol. 3426, Springer-Verlag Berlin, Berlin, pp. 36-57.
- Chella, A., Frixione, M. & Gaglio, S. 1997, 'A cognitive architecture for artificial vision', *Artificial Intelligence*, vol. 89, no. 1-2, pp. 73-111.
- Chella, A., Frixione, M. & Gaglio, S. 2001, 'Conceptual spaces for computer vision representations', *Artificial Intelligence Review*, vol. 16, no. 2, pp. 137-152.
- Cheng, P.C., Lowe, R.K. & Scaife, M. 2001, 'Cognitive science approaches to understanding diagrammatic representations', *Artificial Intelligence Review*, vol. 15, no. 1, pp. 79-94.
- Cleveland, W.S. & McGill, R. 1985, 'Graphical perception and graphical methods for analyzing scientific-data', *Science*, vol. 229, no. 4716, pp. 828-833.
- Gärdenfors, P. 1997, 'Symbolic, conceptual and subconceptual representations', in V. Cantoni, V. di Gesù, A. Setti & D. Tegolo (eds), *Human and Machine Perception: Information Fusion*, Plenum Press, New York, pp. 255-270.
- Gärdenfors, P. 2000, *Conceptual Spaces: The geometry of Thought*, A Bradford Book, MIT Press, Cambridge, Massachusetts.
- Gärdenfors, P. & Williams, M.A. 2001, 'Reasoning about categories in conceptual spaces' *Proceedings of IJCAI2001*, pp.385-392.
- Garner, W.R. 1974, *The processing of Information and Structure*, Erlbaum, Potomac, MD.
- Hegarty, M. 2004, 'Diagrams in the mind and in the world: Relations between internal and external visualizations', *3rd International Conference on Diagrams*, eds A. Blackwell, K. Marriott & A. Shimojima, Cambridge, England, pp. 1-13.
- Hofmann, H.J. 1994a, 'Grain-shape indexes and isometric graphs', *Journal of Sedimentary Research A - Sedimentary Petrology and Processes*, vol. 64, no. 4, pp. 916-920.
- Hofmann, H.J. 1994b, *Grain shape analysis: Including shape entropy function*, <<http://www.eps.mcgill.ca/~hofmann/Shape-entropy.ppt>>.
- Howarth, R.J. 1996, 'Sources for a history of the ternary diagram', *British Journal for the History of Science*, vol. 29, no. 102, pp. 337-356.
- Kullback, S. & Leibler, R.A. 1951, 'On information and sufficiency', *Annals of Mathematical Statistics*, vol. 22, no. 1, pp. 79-86.
- Le Roux, J.P. 2005, 'Grains in motion: A review', *Sedimentary Geology*, vol. 178, no. 3-4, pp. 285-313.
- Pelto, C.R. 1954, 'Mapping of multicomponent systems', *Journal of Geology*, vol. 62, no. 5, pp. 501-511.
- Prosim 2004, *ProSim Ternary Diagram 1.0.3.0* edn, <<http://www.prosim.net/en/resources/download.html>>.
- Rosch, E. 1975, 'Cognitive representations of semantic categories', *Journal of Experimental Psychology-General*, vol. 104, no. 3, pp. 192-233.
- Rossi, C. 2000, 'Bruno de Finetti: the mathematician, the statistician, the economist, the forerunner', paper presented to the *21st Annual Meeting of the International Society for Clinical Biostatistics (ISCB)*, Trent, Italy, Sep 04-08.
- Shannon, C.E., and Weaver, W. 1949, *The Mathematical Theory of Communication*, University of Illinois Press.
- Sneed, E.D. & Folk, R.L. 1958, 'Pebbles in the lower-Colorado-River, Texas: A study in particle morphogenesis', *Journal of Geology*, vol. 66, no. 2, pp. 114-&.
- Todd, J.T. 2004, 'The visual perception of 3D shape', *Trends in Cognitive Sciences*, vol. 8, no. 3, pp. 115-121.
- Weisstein, E.W. 2009, *Simplex*, <<http://mathworld.wolfram.com/Simplex.html>>.